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### TWENTY-SECOND ANNUAL REPORT OF COMMITTEE ON ATOMIC WEIGHTS. DETERMINATIONS PUBLISHED DURING 1914.

By GREGORY PAUL BAXTER.

Received January 21, 1915.

During the past year reports of investigations upon many of the most used and important atomic weights have been published. The work upon carbon, sulfur, iodine, and the radio-elements, uranium and lead, is particularly interesting and significant.

**Oxygen.**—In the report for 1913 some gas density determinations by Scheuer<sup>1</sup> were overlooked. Oxygen was prepared from potassium permanganate and potassium chlorate, and after purification was fractionally distilled. The value obtained for the weight of the normal liter is a few units higher in the sixth figure than the one commonly assumed.

#### WEIGHT OF NORMAL LITER OF O<sub>2</sub>.

From KMnO <sub>4</sub> .		From KClO <sub>3</sub> .	
1.42908	1.42899	1.42889	1.42919
1.42888	1.42922	1.42904	1.42909
1.42924	1.42886	1.42921	1.42895
1.42911	1.42904	1.42898	1.42911
1.42913	1.42906	1.42909	1.42902
1.42889	1.42921	1.42897	1.42914
Mean, 1.42906		Mean, 1.42906	

Mean, 1.42906

<sup>1</sup> *Anzeiger d. k. Akad. Wien*, 50, 378 (1913).

**Nitrogen.**—Scheuer also found the density of nitric oxide, using carefully purified gas.

## WEIGHT OF NORMAL LITER OF NO.

1.34002	1.34008	1.34019	1.34014
1.34028	1.34013	1.33994	1.34031
1.34022	1.34015	1.34021	1.34017
1.34004	1.34017	1.34023	1.33995
Mean, 1.34014			

Computed by the method of limiting densities,  $N = 14.004$ .

**Chlorine.**—For hydrochloric acid Scheuer obtained the following results:

## WEIGHT OF NORMAL LITER OF HCl.

1.63925	1.63931	1.63910	1.63897
1.63894	1.63908	1.63929	1.63923
1.63912	1.63925	1.63918	1.63913
1.63915	1.63937	1.63915	1.63920
Mean, 1.63917			

Computed by the method of limiting densities,  $HCl = 36.468$  and  $Cl = 35.460$ .

**Neon.**—Leduc<sup>1</sup> fractionated neon with cocoanut charcoal at low temperatures, and, using a 600 cc. globe, in two experiments found the gas to be 0.6964 and 0.6958 times as heavy as air at 0°. Allowance for a slight amount of impurity was made by taking the density as 0.695. From this value and the one 0.06948 for hydrogen, the atomic weight of neon is calculated to be 20.15 ( $H = 1.0075$ ).

**Carbon.**—Timmermans<sup>2</sup> prepared propane (a) by the action of propyl iodide on "sodium ammonium" and (b) by the action of sodium on propyl cyanide. The gas was liquefied and fractionally distilled. The weights of the normal liter, found by the globe method, are given below:

	(a)			
Volume of globe	564.88 cc.	455.77 cc.	351.91 cc.	Mean.
	2.02021	2.01928	2.01744	2.01898
	(2.02287)	2.01807	2.01970	2.01888
	2.02070	2.01940	(2.02258)	2.02005
	2.01864	2.02110	2.02125	2.02033
	Mean, 2.01985	2.01946	2.01946	2.01956
	(b)			
	2.01809	(2.01580)	2.02127	2.01968
	2.01807	2.02161	2.01947	2.01972
	2.01928	2.01933	(2.02352)	2.01930
	Mean, 2.01848	2.02047	2.02037	2.01957
	General mean, 2.01956			

<sup>1</sup> *Compt. rend.*, 158, 864 (1914).

<sup>2</sup> *Ibid.*, 158, 789 (1914).

The other data necessary for computing the molecular weight of propane are now being determined in Geneva.

Richards and Hoover<sup>1</sup> titrated weighed amounts of sodium carbonate, which had been fused in a current of dry carbon dioxide, against weighed amounts of a solution of hydrobromic acid which had been standardized both by titration against the purest silver and by weighing the silver bromide produced. The hydrobromic acid solution was found to be equivalent to 8.3113 g. in vacuum of silver bromide per 100.000 g. in vacuum of solution. Furthermore, the solution of sodium bromide resulting from the neutralization was compared with weighed amounts of silver. The weights in the table are corrected to the vacuum standard and the following atomic weights are assumed:

$$\text{Ag} = 107.880; \text{Br} = 79.916; \text{Na} = 22.995.$$

Weight Na <sub>2</sub> CO <sub>3</sub> .	Weight HBr solution.	At. wt. C.	Weight Ag.	At. wt. C.
4.75555	202.744	12.008	9.68023	12.005
4.80081	204.673	12.009	9.77222	12.007
4.88936	208.457	12.004	9.95301	12.001
5.63157	240.119	11.997	11.46316	12.008
4.49516	191.646	12.006	9.15003	12.007
4.86256	207.307	12.008	9.89811	12.004
		Mean,		Mean,
		12.005		12.005

**Sulfur.**—Richards and Hoover<sup>2</sup> also neutralized weighed amounts of sodium carbonate with dilute sulfuric acid. The solution was evaporated to small bulk and transferred to a weighed quartz flask, and after evaporation of the solution to dryness the residual sodium sulfate was fused. Vacuum weights are given and the atomic weights of carbon and sodium are assumed to be 12.005 and 22.995, respectively.

Weight Na <sub>2</sub> CO <sub>3</sub> .	Weight Na <sub>2</sub> SO <sub>4</sub> .	At. wt. S.
5.25191	7.03829	32.058
4.50977	6.04382	32.060
5.04233	6.75737	32.057
3.67340	4.92304	32.063
4.18724	5.61151	32.059
4.55100	6.09916	32.062
		Mean,
		32.060

**Silver.**—Richards and Cox<sup>3</sup> have published further details of their experiments which show that lithium perchlorate, as prepared by Richards and Willard in their work upon the atomic weights of lithium and silver, is free from moisture. The question as to whether the atomic weight of silver is as low as 107.87 certainly is still an open one.

<sup>1</sup> THIS JOURNAL, 37, 95 (1915).

<sup>2</sup> *Ibid.*, 37, 108 (1915).

<sup>3</sup> *Ibid.*, 36, 819 (1914).

**Iodine.**—Bates and Vinal<sup>1</sup> made a very careful comparison of the silver and iodine coulometers, using them in series. The silver coulometers were of Smith's "new type" and the "porous pot" varieties; the iodine coulometers of the form devised by Washburn and Bates, the iodine set free being titrated by means of an arsenite solution which had been standardized with weighed amounts of iodine. In the following tables each weight is the mean from several coulometers run in series. Vacuum corrections are applied to the weights of iodine and presumably to the weights of silver.  $Ag = 107.880$ .

Weight silver.	Weight iodine.	At. wt. I.
(4.10582)	(4.82959)	(126.897)
(4.10469)	(4.82862)	(126.907)
4.09903	4.82224	126.914
4.39711	5.17273	126.910
4.10523	4.82851	126.887
4.12310	4.84942	126.884
4.10475	4.82860	126.904
4.18424	4.92130	126.883
4.10027	4.82247	126.882
4.10516	4.82844	126.887

Mean of last eight experiments, 126.894

Bates and Vinal point out that this result is subject to correction for inclusions of electrolyte by the silver deposits. While Rosa, Vinal and McDaniel<sup>2</sup> have recently expressed the opinion that highly crystalline deposits from the purest silver nitrate solutions do not contain significant inclusions, Richards and Anderegg<sup>3</sup> have found the inclusions of electrolyte to amount to from 0.004 to 0.035% of the weight of silver, according to conditions. These proportions are somewhat larger than those recently found by Hulett. Richards and Anderegg point out that all atomic weights determined electrolytically in the past by comparison with silver coulometers are therefore in error, by amounts impossible of accurate estimation because of the irregular nature of the inclusions. If, however, an average correction, as found by Richards and Anderegg, of 0.015% is applied to Bates and Vinal's results, the atomic weight of iodine becomes 126.913, a value in much better agreement with the most recent gravimetric determination by Baxter, 126.932.

Guichard<sup>4</sup> also has compared the atomic weight of iodine directly with that of oxygen by decomposing known weights of iodine pentoxide and weighing the products. The iodine pentoxide was prepared from iodic acid and was dried at 240° in a vacuum, the iodine was condensed in the decomposition apparatus and weighed, the oxygen was collected and weighed

<sup>1</sup> THIS JOURNAL, 36, 916 (1914); Bureau of Standards, *Sci. Paper* No. 218.

<sup>2</sup> Bureau of Standards, *Sci. Paper* No. 220 (1914).

<sup>3</sup> THIS JOURNAL, 37, 7 (1915).

<sup>4</sup> *Compt. rend.*, 159, 185 (1914).

as copper oxide. The various substances were all weighed in exhausted vessels against counterpoises, with the following results:

Weight I <sub>2</sub> O <sub>5</sub> .	Weight I <sub>2</sub> .	Weight O <sub>8</sub> .	Weight I <sub>2</sub> + weight O <sub>8</sub> .	Difference.
5.18051	3.93910	1.24183	5.18093	—0.00042
7.80826	5.93688	1.87138	7.80826	0.00000
9.33281	7.09689	2.23637	9.33326	—0.00045
6.23324	4.73922	1.49333	6.23255	+0.00069
7.08076	5.38441	1.69714	7.08155	—0.00079

The atomic weight of iodine is computed in three ways.

$40 \frac{\text{wt. I}_2}{\text{wt. O}_8}$	$40 \frac{\text{wt. I}_2}{\text{wt. I}_2\text{O}_5 - \text{wt. I}_2}$	$40 \frac{\text{wt. I}_2\text{O}_5 - \text{wt. O}_8}{\text{wt. O}_8}$
126.88	126.92	126.87
126.90	126.90	126.90
126.94	126.96	126.93
126.94	126.89	126.96
126.91	126.96	126.89
Mean, 126.91	Mean, 126.93	Mean, 126.91
	General mean, 126.92	

No correction is made for moisture retained by the iodine pentoxide. It is noteworthy that the average deficiency of the weights of the products is 0.0028%, a quantity in close agreement with the moisture content of iodine pentoxide dried at the same temperature, as found by Baxter and Tilley,<sup>1</sup> 0.0023%. The second mean is raised, the third lowered by applying this correction. The general mean is not appreciably affected, however.

Guye and Germann<sup>2</sup> investigated iodine and silver for occluded gases. One gram of iodine was found to contain 0.000013 g. air. One gram of silver, which had been fused in a current of hydrogen, yielded 0.000034 g. carbon monoxide and 0.000008 g. moisture. Neither material was of the highest purity.

**Barium.**—De Coninck<sup>3</sup> determined the proportion of carbon dioxide in barium carbonate by dissolving the salt in acid. The average of five experiments gave Ba = 137.4.

**Nickel.**—De Coninck with Gerard<sup>4</sup> also has determined the atomic weight of nickel, by reducing the hydrated oxalate in a current of hydrogen.

Weight NiC <sub>2</sub> O <sub>4</sub> ·2H <sub>2</sub> O.	Weight Ni.	At. wt. Ni.
0.4300	0.1380	58.60
0.4730	0.1517	58.54
0.3440	0.1104	58.60
0.3869	0.1241	58.55
0.5161	0.1656	58.58
		Mean, 58.57

<sup>1</sup> THIS JOURNAL, 31, 212 (1909).

<sup>2</sup> *Compt. rend.*, 159, 225 (1914).

<sup>3</sup> *Rev. gén. chim.*, 16, 245 (1914).

<sup>4</sup> *Compt. rend.*, 158, 1345 (1914).

**Cadmium.**—Baxter and Hartmann<sup>1</sup> have determined the cadmium content of fused cadmium chloride by direct electrolysis with a mercury cathode.

Weight CdCl <sub>2</sub> .	Weight Cd.	At. wt. Cd.
Preliminary Series.		
6.08570	3.73181	112.426
4.20489	2.57863	112.443
5.36203	3.28817	112.436
7.50512	4.60221	112.426
6.71591	4.11839	112.435
5.91556	3.62763	112.438
5.49323	3.36805	112.387
3.14416	1.92791	112.408
7.58705	4.65173	112.381
2.26738	1.39006	112.359
5.93501	3.63962	112.443
6.04122	3.70490	112.454
4.07400	2.49821	112.425
		Mean, 112.420
Final Series.		
9.00004	5.51879	112.419
6.56891	4.02808	112.423
7.12956	4.37174	112.414
8.57291	5.25679	112.415
7.76294	4.76011	112.413
		Mean, 112.417

This result is considerably higher than the value recently found by Hulett and his collaborators, but in close agreement with that previously obtained by Baxter, Hines and Frevert.

**Mercury.**—H. B. Baker and Watson<sup>2</sup> synthesized mercuric bromide from the elements. An excess of bromine retained by the halide was quantitatively estimated. Vacuum weights are given in the table. Br = 79.92.

Weight Hg.	Weight HgBr <sub>2</sub> .	At. wt. Hg.
4.88190	8.77274	200.56
6.74796	12.12646	200.54
4.79430	8.61640	200.50
4.52480	8.13048	200.58
4.78892	8.60534	200.57
5.52880	9.93406	200.61
4.40148	7.90886	200.59
4.52338	8.12794	200.58
5.44704	9.78683	200.62
		Mean, 200.57

<sup>1</sup> THIS JOURNAL, 37, 113 (1915).

<sup>2</sup> J. Chem. Soc., 105, 2530 (1914).

This value is 0.2 unit higher than the result reported a year ago by Taylor and Hulett, but confirms very closely the recent work of Easley.

**Vanadium.**—Briscoe and Little<sup>1</sup> synthesized vanadyl chloride from chloride and vanadium trioxide, and separated the product into four portions, A, B, C, D, by fractional distillation, with rejection of the two extreme fractions. Two of the four main fractions, C and D, were further fractionated in the same way into weighed bulbs which were sealed while exhausted. Analysis was effected by decomposition with ammonia, acidification, and comparison with silver. In two cases the silver chloride was collected and weighed. Vacuum corrections are applied. Ag = 107.880; Cl = 35.457.

Fraction.	Weight VOCl <sub>2</sub> .	Weight Ag.	At. wt. V.
C 2.....	3.98162	7.43451	50.957
C 3.....	5.63804	10.52736	50.958
C 5.....	3.70889	6.92632	50.931
C 6.....	3.88228	7.24884	50.962
D 1.....	3.28485	6.13377	50.950
D 2.....	3.45920	6.45900	50.959
D 5.....	3.17088	5.92089	50.951
D 6.....	3.63990	6.79670	50.951
D 7.....	3.23254	6.20682	50.927

Mean, 50.950

Fraction.	Weight VOCl <sub>2</sub> .	Weight AgCl.	At. wt. V.
C 4.....	4.42279	10.97187	50.968
D 8.....	2.62557	6.51459	50.936

Mean, 50.952

This result agrees closely with the value recently found by McAdam, 50.96.

**Tellurium.**—Still another attempt has been made to separate from tellurium a related element of higher atomic weight. Dennis and Anderson<sup>2</sup> prepared hydrogen telluride from aluminum telluride by the action of dilute hydrochloric acid, and, after being dried, the gas was condensed with liquid air and fractionally distilled. Each fraction was converted to metallic tellurium at a high temperature, and, after being weighed, was oxidized with nitric acid and the dioxide was weighed. The results varied irregularly, 31 out of 45 falling between 127.3 and 127.8. Another specimen of hydride resulted from the electrolysis of a phosphoric acid solution with tellurium cathodes. The fractions of this preparation were caught in nitric acid and after being weighed as the dioxide were dissolved and titrated with potassium permanganate. The average of thirty-seven results, from 22 fractions, between 127.76 and 127.36, is 127.50. The authors consider that there is no evidence of the existence of divi-tellurium.

<sup>1</sup> *J. Chem. Soc.*, 105, 1310 (1914).

<sup>2</sup> *THIS JOURNAL*, 36, 882 (1914).

**Radioactive Lead.**—Four determinations of the atomic weight of radioactive lead appeared within a short period. Richards and Lemberg<sup>1</sup> analyzed lead chloride obtained from minerals consisting largely of radioactive substances. The radioactivity due to  $\beta$ -rays was found several weeks after crystallization.

Source.	Radioactivity referred to UO <sub>2</sub> .	Wt. PbCl <sub>2</sub> .	Wt. Ag.	Wt. AgCl.	At. wt.
Colorado carnotite.....	21.8	3.87082	3.00984	.....	206.56
		5.57331	4.33300	.....	206.60
		5.49412	4.27157	.....	206.59
		3.87082	.....	3.99879	206.59
		5.57331	.....	5.75707	206.61
		5.49412	.....	5.67573	206.58
					Mean, 206.59
Ceylon thorianite (Boltwood) 5.0		3.88228	3.01600	.....	206.81
		4.05550	3.15061	.....	206.81
		4.05168	3.14788	.....	206.79
		3.88228	.....	4.00703	206.84
		4.05168	.....	4.18265	206.78
					Mean, 206.81
Ceylon thorianite (Miner).. 3.8		2.80814	2.18162	.....	206.81
		2.80814	.....	2.89816	206.85
					Mean, 206.83
Joachimsthal pitchblende.... 5.7		3.95052	3.07209	.....	206.54
		2.95726	2.29951	.....	206.56
		2.95726	.....	3.05475	206.61
					Mean, 206.57
English pitchblende..... 3.5		4.05702	3.15153	.....	206.84
		4.05702	.....	4.18670	206.88
					Mean, 206.86
No. Carolina uraninite.... 13.2		2.01795	1.56952	.....	206.47
		2.01795	.....	2.08663	206.32
					Mean, 206.36

By the same method Richards and Lemberg found in preliminary experiments the atomic weight of common lead to be 207.15.<sup>2</sup>

Lead from Colorado carnotite was found to give a spectrum exactly like that of ordinary lead so far as could be determined.

<sup>1</sup> THIS JOURNAL, 36, 1329 (1914).

<sup>2</sup> Baxter and Grover have recently obtained the value 207.20 by analysis of both bromide and chloride. *Carnegie Institution Yearbook* No. 13. The details of this work will be published shortly.

Hönigschmid and Mme. Horowitz<sup>1</sup> utilized lead separated from pitchblende, and employed the usual chloride method. Their average result is between those of the two pitchblende samples examined by Richards and Lembert.

Wt. PbCl <sub>2</sub> .	Wt. AgCl.	Wt. Ag.	At. wt.
1.97691	2.04118	.....	206.733
3.04040	3.13910	.....	206.746
3.05981	3.15939	.....	206.724
3.30285	3.41015	.....	206.740
3.33164	3.44013	.....	206.719
3.22459	3.32923	.....	206.749
3.33164	.....	2.58888	206.748
3.22459	.....	2.50586	206.730
3.79447	.....	2.71545	206.744

Mean, 206.735

M. Curie<sup>2</sup> also examined active lead from various minerals. After elaborate purification the metal was fused under cyanide and weighed. Then it was dissolved in nitric acid in quartz and the nitrate was dried at 150° before being weighed.

Source.	At. wt.	At. wt. after repurification.	Mean.
Pitchblende.....	206.60	206.68	206.64
Carnotite.....	206.38	206.34	206.36
Yttriotantalite.....	206.49	{ 206.59	206.54
		{ 206.55	
Monazite.....	207.06	207.10	207.08
Galena.....	206.98	207.04	207.01

The results for pitchblende and carnotite, when allowance is made for the fact that ordinary lead from galena gave a low value, agree very well with those of Richards and Lembert, and Hönigschmid. It is noteworthy that monazite lead seems to possess a *higher* atomic weight than ordinary lead.

Soddy and Hyman<sup>3</sup> by a comparative method also found lead from thorite to have an atomic weight 1.3 units *higher* than that of ordinary lead.

Fayans<sup>4</sup> has discussed the bearing of Richards and Lembert's results upon the present theories of the relation of lead to radioactive disintegration.

**Uranium.**—Hönigschmid<sup>5</sup> has analyzed uranium tetrabromide made by the action of bromine upon a mixture of the dioxide and carbon. In Series I the salt was sublimed, fused and allowed to solidify in an atmosphere containing bromine vapor; in Series II the salt was finally fused

<sup>1</sup> *Z. Elekt.*, 20, 319, 457 (1914); *Compt. rend.*, 158, 1796.

<sup>2</sup> *Compt. rend.*, 158, 1676 (1914).

<sup>3</sup> *J. Chem. Soc.*, 105, 1402 (1914).

<sup>4</sup> *Z. Elect.*, 20, 449 (1914).

<sup>5</sup> *Ibid.*, 20, 452 (1914); *Compt. rend.*, 158, 2004.

Series I.		
Weight UBr.	Weight AgBr.	At. wt. U.
3.40006	4.57930	238.08
3.91408	5.27180	238.06
4.52766	6.09791	238.09
5.13497	6.91566	238.10
3.73118	5.02536	238.07
4.68978	6.34652	238.06
3.28480	4.42400	238.09
3.91067	5.26715	238.06

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Mean, 238.08

Weight UBr.	Weight Ag.	At. wt. U.
4.52766	3.50286	238.10
5.13497	3.97256	238.12
4.68978	3.62849	238.07
3.28480	2.54131	238.10
3.91067	3.02567	238.07
4.16254	3.22038	238.10

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Mean, 238.09

Series II.		
Weight UBr.	Weight AgBr.	At. wt. U.
2.89549	3.89900	238.18
2.82823	3.80830	238.20
3.95050	5.31979	238.17
3.57277	4.81110	238.17
3.91948	5.27803	238.17
3.77074	5.07761	238.18
4.31113	5.80562	238.15
3.33325	4.48823	238.21
5.14622	6.93002	238.16
5.05193	6.80279	238.18
5.33502	7.18387	238.19
4.84910	6.52962	238.19
5.24735	7.06581	238.19
4.56095	6.14204	238.15

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Mean, 238.18

Weight UBr.	Weight Ag.	At. wt. U.
3.91948	3.03190	238.18
3.77074	2.91689	238.17
4.31113	3.33505	238.15
5.14622	3.98089	238.17
5.05193	3.90789	238.18
5.33502	4.12698	238.17
4.84910	3.75106	238.17
5.24735	4.05913	238.17
4.56095	3.52825	238.16

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Mean, 238.17

and cooled in nitrogen. The results of Series II are preferred since the salt used in Series I probably contained free bromine. Analysis was effected by comparison with weighed amounts of silver and by weighing the silver bromide formed. Weights are corrected to vacuum.

**Neoytterbrium.**—Blumenfeld and Urbain<sup>1</sup> have purified the nitrate of this element by fractionation, and give its atomic weight as 173.54. This is higher than that of Auer von Welsbach's "aldebaranium," 173.00.

The atomic weights of thorium, cerium, and beryllium have been discussed from the standpoint of transparency to X-rays by Benoist and Copaux,<sup>2</sup> that of nebulium by Bourget, Fabry and Buisson<sup>3</sup> and Nicholson,<sup>4</sup> the relations between the atomic weights by Loring.<sup>5</sup>

CAMBRIDGE, MASS.

[CONTRIBUTION FROM THE RESEARCH LABORATORY OF THE GENERAL ELECTRIC COMPANY.]

## THE DISSOCIATION OF HYDROGEN INTO ATOMS.

[PART II.]

### CALCULATION OF THE DEGREE OF DISSOCIATION AND THE HEAT OF FORMATION.

BY IRVING LANGMUIR.

Received December 21, 1914.

The first part of this paper<sup>6</sup> dealt with experiments on the heat losses from fine tungsten wires in hydrogen at pressures ranging from 0.010 mm. up to atmospheric pressure. The results were given in a series of tables, and form the basis for the present calculation of the degree of dissociation and the heat of formation of molecular hydrogen.

The previous calculations of the dissociation of hydrogen<sup>7</sup> were based on the use of the equation

$$(3)^8 \quad W_D = SDq_1c_1$$

as described in Part I. The equation itself rests on a firm foundation, but the assumptions previously made in applying it to experimental results are open to serious criticism.

In the first place, it was assumed that the diffusion coefficient  $D$  could be calculated with sufficient accuracy from some equations of the kinetic

<sup>1</sup> *Compt. rend.*, 159, 325 (1914).

<sup>2</sup> *Ibid.*, 158, 689, 859 (1914).

<sup>3</sup> *Compt. rend.*, 158, 1017 (1914).

<sup>4</sup> *Ibid.*, 158, 1322 (1914).

<sup>5</sup> *Chemical News*, 108, 188, 247, 305; 109, 169; 110, 25 (1914).

<sup>6</sup> Langmuir and Mackay, *THIS JOURNAL*, 36, 1708 (1914).

<sup>7</sup> *Ibid.*, 34, 860 (1912). This paper will hereafter be referred to as the "paper of 1912."

<sup>8</sup> For the derivation of this and some of the following equations, see Part I, or the "paper of 1912." The numbers of the equations are the same as those used in Part I.